

Temari: GLOBAL

1.-

$$r: \begin{cases} 2x - y + 3z = 2 \\ x + z + 1 = 0 \end{cases}$$

a)

$$\vec{v}_r = (2, -1, 3) \wedge (1, 0, 1) = \begin{vmatrix} i & 2 & 1 \\ j & -1 & 0 \\ k & 3 & 1 \end{vmatrix} = -i + 3j + 0k + k - 0i - 2j = \underline{\underline{(-1, 1, 1)_{\langle i, j, k \rangle}}} \bullet$$

b)

$$s \parallel r \rightarrow \vec{v}_s \parallel \vec{v}_r \Rightarrow \vec{v}_s = (-1, 1, 1) \rightarrow s: \underline{\underline{\frac{x-1}{-1} = \frac{y}{1} = \frac{z-1}{1}}} \bullet$$

2.-

$$X \cdot A + B = C$$

a)

$$X \cdot A + B = C \rightarrow X \cdot A = C - B \rightarrow \underline{\underline{X = (C - B) \cdot A^{-1}}} \bullet$$

b)

$$X = (C - B) \cdot A^{-1} = \left[\begin{pmatrix} 3 & 1 \\ 1 & -1 \end{pmatrix} - \begin{pmatrix} 1 & 1 \\ -2 & 1 \end{pmatrix} \right] \cdot \begin{pmatrix} 1 & -2 \\ -1 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 2 & 0 \\ 3 & -2 \end{pmatrix} \cdot \begin{pmatrix} 1 & -2 \\ -1 & 1 \end{pmatrix}^{-1} = *$$

$$\begin{pmatrix} 1 & -2 \\ -1 & 1 \end{pmatrix}^{-1} : |A| = 1 - 2 = -1, A^T = \begin{pmatrix} 1 & -1 \\ -2 & 1 \end{pmatrix} \rightarrow A^{-1} = \frac{1}{-1} \begin{pmatrix} 1 & +2 \\ +1 & 1 \end{pmatrix} = \begin{pmatrix} -1 & -2 \\ -1 & -1 \end{pmatrix}$$

$$* = \begin{pmatrix} 2 & 0 \\ 3 & -2 \end{pmatrix} \cdot \begin{pmatrix} -1 & -2 \\ -1 & -1 \end{pmatrix} = \underline{\underline{\begin{pmatrix} -2 & -4 \\ -1 & -2 \end{pmatrix}}} \bullet$$

3.-

$$f(x) = a \cdot (1 - x^2) \text{ i } g(x) = \frac{x^2 - 1}{a} \quad a > 0.$$

a)

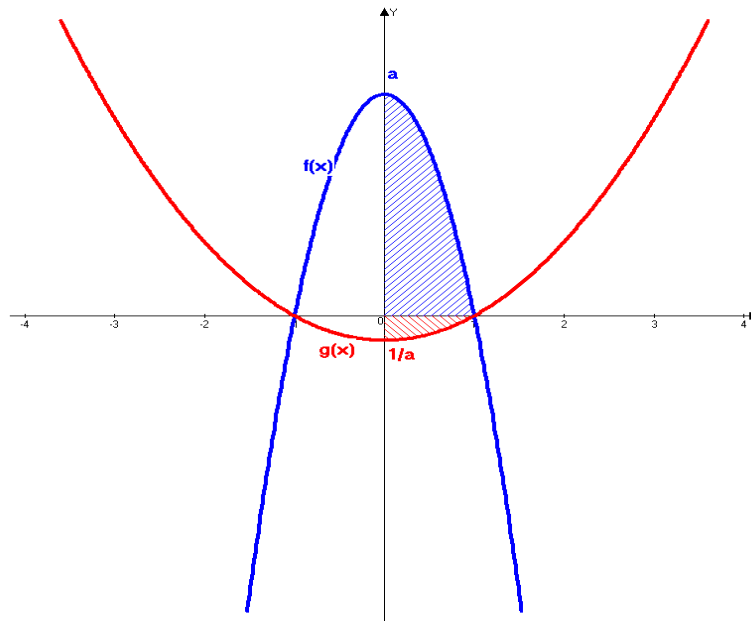
$$f(x) = a \cdot (-x^2 + 1) \text{ i } g(x) = \frac{1}{a} \cdot (x^2 - 1) \text{ amb } a > 0$$

$$\text{Punts de tall: } f(x) = g(x) \Rightarrow a \cdot (-x^2 + 1) = \frac{1}{a} \cdot (x^2 - 1) \rightarrow -a^2 x^2 + a^2 = x^2 - 1$$

$$\rightarrow -a^2 x^2 - x^2 = -1 - a^2 \rightarrow x^2 \cdot (-a^2 - 1) = -1 - a^2 \rightarrow x^2 = 1 \rightarrow x = \pm 1$$

$$\text{i a més } f(\pm 1) = g(\pm 1) = 0.$$

Tenim dues paràboles, una còncava i una altra convexa, que tenen per zeros $x = \pm 1$



$$\begin{aligned} 2 \int_0^1 [f(x) + (-g(x))] dx &= 2 \int_0^1 [f(x) - g(x)] dx = 2 \cdot \int_0^1 \left[a \cdot (1-x^2) - \frac{1}{a} \cdot (x^2-1) \right] dx = \\ &= 2 \cdot \int_0^1 \left[a \cdot (1-x^2) + \frac{1}{a} \cdot (-x^2+1) \right] dx = 2 \cdot \int_0^1 \left(a + \frac{1}{a} \right) \cdot (-x^2+1) \cdot dx = \\ &= 2 \cdot \left(a + \frac{1}{a} \right) \cdot \left[-\frac{x^3}{3} + x \right]_0^1 = 2 \cdot \left(a + \frac{1}{a} \right) \cdot \left(-\frac{1}{3} + 1 \right) = 2 \cdot \left(a + \frac{1}{a} \right) \cdot \frac{2}{3} = \frac{4}{3} \cdot \left(\frac{a^2+1}{a} \right) \bullet \end{aligned}$$

b)

Superfície en funció de a : $S(a) = \frac{4}{3} \cdot \left(\frac{a^2+1}{a} \right)$

$$S'(a) = \frac{4}{3} \cdot \frac{2a \cdot a - (a^2+1) \cdot 1}{a^2} = \frac{4}{3} \cdot \frac{a^2-1}{a^2} \rightarrow S'(a) = 0 \Rightarrow a^2-1=0 \rightarrow a = \pm 1$$

$$S''(a) = \frac{4}{3} \cdot \frac{2a \cdot a^2 - (a^2-1) \cdot 2a}{a^4} \rightarrow \begin{cases} S''(1) = \frac{(+)-0}{(+)} > 0 \Rightarrow \underline{\underline{\text{mínim}}} \bullet \\ S''(-1) = \frac{(-)-0}{(+)} < 0 \rightarrow \text{màxim} \end{cases}$$

4.-

$$\begin{cases} x + 2y - az = -3 \\ 2x + (a-5)y + z = 4a + 2 \\ 4x + (a-1)y - 3z = 4 \end{cases}$$

a)

$$(A:\bar{A}) = \begin{pmatrix} 1 & 2 & -a & \vdots & -3 \\ 2 & a-5 & 1 & \vdots & 4a+2 \\ 4 & a-1 & -3 & \vdots & 4 \end{pmatrix}$$

$$\det A = \begin{vmatrix} 1 & 2 & -a \\ 2 & a-5 & 1 \\ 4 & a-1 & -3 \end{vmatrix} = -3a + 15 - 2a^2 + 2a + 8 + 4a^2 - 20a - a + 1 + 12 = 2a^2 - 22a + 36$$

$$2a^2 - 22a + 36 = 0 \rightarrow a^2 - 11a + 18 = 0 \rightarrow \begin{cases} a = 2 \\ a = 9 \end{cases}$$

* Si $a \neq 2$ i $a \neq 9 \rightarrow \det A \neq 0 \rightarrow \text{rang} A = \text{rang} \bar{A} = 3 \rightarrow C..D.(\text{no interessa})$

* Si $a = 2$ o $a = 9$: $\det A = 0 \Rightarrow \text{rang} A < 3$ no pot ser C.D. •

** $a = 2$

$$\begin{pmatrix} 1 & 2 & -2 & \vdots & -3 \\ 2 & -3 & 1 & \vdots & 10 \\ 4 & 1 & -3 & \vdots & 4 \end{pmatrix} \rightarrow \begin{vmatrix} 1 & 2 \\ 2 & -3 \end{vmatrix} \neq 0 \rightarrow \text{rang} A = 2 \quad \begin{vmatrix} 1 & 2 & -3 \\ 2 & -3 & 10 \\ 4 & 1 & 4 \end{vmatrix} = 0$$

i com $C_3 = -C_2 \rightarrow$ tots els menors d'ordre 3 de \bar{A} tenen determinant zero \rightarrow
, $\text{rang} \bar{A} = 2$ COMPATIBLE INDETERMINAT.

b)

$$\begin{cases} x + 2y - az = -3 \\ 2x + (a-5)y + z = 4a + 2 \\ 4x + (a-1)y - 3z = 4 \end{cases} \rightarrow \begin{cases} 1 - 6 + a = -3 \\ 2 - (a-5)3 - 1 = 4a + 2 \\ 4 - (a-1)3 + 3 = 4 \end{cases} \rightarrow \begin{cases} a = 2 \\ a = 2 \\ a = 2 \end{cases}$$

però per aquest valor de a , el sistema és compatible indeterminat, per tant
la solució donada no pot ser solució única. \rightarrow no existeix tal valor de a . •

5.-

$$r_1: \frac{x-2}{1} = \frac{y-3}{2} = \frac{z-1}{-2} \quad r_2: \frac{x+3}{2} = \frac{y+1}{1} = \frac{z+1}{2}$$

a)

$$\vec{v}_{r_1} \cdot \vec{v}_{r_2} = (1, 2, -2) \cdot (2, 1, 2) = 2 + 2 - 4 = 0 \rightarrow \vec{v}_{r_1} \perp \vec{v}_{r_2} \rightarrow r_1 \perp r_2 \bullet$$

b)

$$r_1: \frac{x-2}{1} = \frac{y-3}{2} = \frac{z-1}{-2} \rightarrow \begin{cases} 2x - 4 = y - 3 \\ -2x + 4 = z - 1 \end{cases}$$

$$r_2: \frac{x+3}{2} = \frac{y+1}{1} = \frac{z+1}{2} \rightarrow \begin{cases} x + 3 = 2y + 2 \\ z + 1 = 2y + 2 \end{cases}$$

$$r_1 \cap r_2: \begin{cases} 2x - 1 = y \\ -2x + 5 = z \\ \frac{x+1}{2} = y \\ z = 2y + 1 \end{cases} \rightarrow 2x - 1 = \frac{x+1}{2} \rightarrow \underline{\underline{x=1}} \rightarrow \underline{\underline{y=1}} \quad \underline{\underline{z=3}} \bullet$$

6.-

$$f(x) = x^2 \cdot e^{-ax}, \quad a \neq 0$$

a)

$$f(x) = x^2 \cdot e^{-ax}$$

$$f'(x) = 2x \cdot e^{-ax} + x^2 \cdot e^{-ax}(-a) = 2x \cdot e^{-ax} - a \cdot x^2 \cdot e^{-ax} = e^{-ax} \cdot (2x - ax^2)$$

$$f'(2) = e^{-a \cdot 2} \cdot (2 \cdot 2 - a \cdot 4) = 0 \rightarrow 2 \cdot 2 - a \cdot 4 = 0 \rightarrow \underline{\underline{a=1}} \bullet$$

b)

$$f(x) = x^2 \cdot e^{-2x}$$

$$f'(x) = 2x \cdot e^{-2x} + x^2 \cdot e^{-2x}(-2) = 2x \cdot e^{-2x} - 2 \cdot x^2 \cdot e^{-2x} = e^{-2x} \cdot (2x - 2x^2) = 0$$

$$\rightarrow 2x - 2x^2 = 0 \rightarrow 2x \cdot (1 - x) = 0 \rightarrow \begin{cases} x = 0 \\ x = 1 \end{cases}$$

$$f''(x) = e^{-2x} \cdot (-2) \cdot (2x - 2x^2) + e^{-2x} \cdot (2 - 4x)$$

$$\begin{cases} f''(0) = e^0 \cdot (-2) \cdot 0 + e^0 \cdot (2 - 0) = 0 + 2 > 0 \rightarrow \underline{\underline{(0,0) \text{ mínim}}} \bullet \end{cases}$$

$$\begin{cases} f''(1) = e^{-2} \cdot (-2) \cdot 0 + e^{-2} \cdot (2 - 4) = \frac{1}{e^2} \cdot (-2) < 0 \rightarrow \underline{\underline{(1, 1/e^2) \text{ màxim}}} \bullet \end{cases}$$