1.-

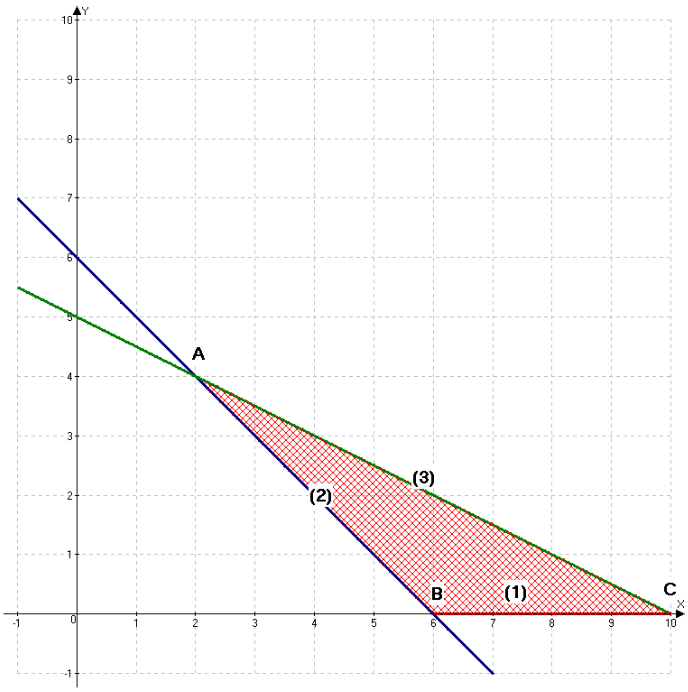
$$\left.\begin{array}{l} x: \text{preu del primer immoble} \\ y: \text{preu del segon immoble} \\ z: \text{preu del tercer immoble} \end{array}\right\}$$

$$\left.\begin{array}{l} x+y+z=2 \\ 0'20x+0'50y+0'25z=0'6 \\ 0'80x+0'90y+0'85z=1'7 \end{array}\right\} \rightarrow \underline{x=2-y-z} \Rightarrow \left.\begin{array}{l} 0'2\cdot(2-y-z)+0'5y+0'25z=0'6 \\ 0'8\cdot(2-y-z)+0'9y+0'85z=1'7 \end{array}\right\} \rightarrow$$

$$\rightarrow \left.\begin{array}{l} 0'3y+0'05z=0'2 \\ 0'1y+0'05z=0'1 \end{array}\right\} \rightarrow \text{restant}: \quad 0'2y=0'1 \Rightarrow \underline{\underline{y=0'5 \text{ milions}, \rightarrow z=1 \text{, milió}}}$$

$$\rightarrow \underline{\underline{x=0'5 \text{ milions}}} \bullet$$

2.-a)



$$\text{Equació explícita de la recta} :\rightarrow \quad y=mx+n$$

$$(1) \quad \text{Eix } OX \rightarrow y=0\bullet$$

$$(2) \rightarrow \left\{\begin{array}{l}(6,0) \\ (2,4)\end{array}\right\} \rightarrow \left\{\begin{array}{l}0=6m+n \\ 4=2m+n\end{array}\right\} \Rightarrow \left\{\begin{array}{l}m=-1 \\ n=6\end{array}\right\} \rightarrow y=-x+6\bullet$$

$$(3) \rightarrow \left\{\begin{array}{l}(10,0) \\ (0,5)\end{array}\right\} \rightarrow \left\{\begin{array}{l}0=10m+n \\ 5=0m+n\end{array}\right\} \Rightarrow \left\{\begin{array}{l}m=-1/2 \\ n=5\end{array}\right\} \rightarrow y=-\frac{1}{2}x+5\bullet$$

$$\left.\begin{array}{l} Per\ sobre\ de\ (1) \\ Per\ sobre\ de\ (2) \\ Per\ sota\ de\ (3) \end{array}\right\} \Rightarrow \begin{cases} y \geq 0 \\ y \geq -x+6 \\ y \leq -\frac{1}{2}x+5 \end{cases} \bullet$$

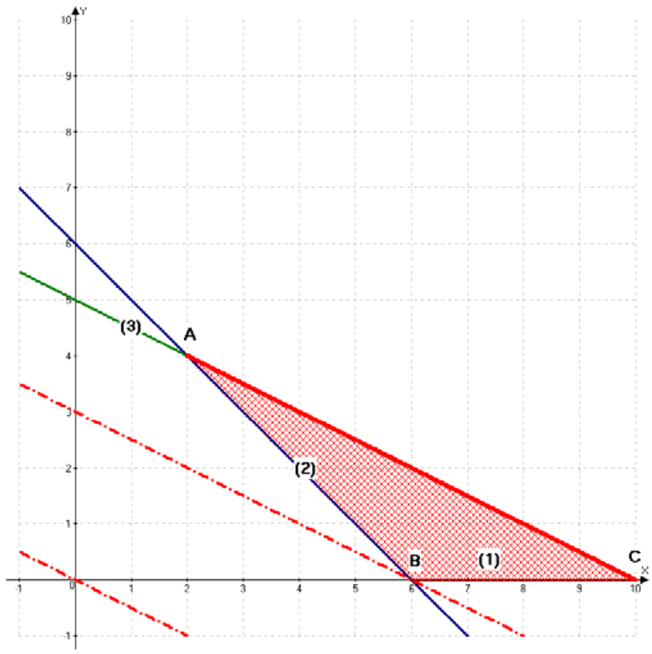
b)

Vèrtexs de la regió	$A(2,4)$	$B(6,0)$	$C(10,0)$
$z = x+2y$	$z=2+2\cdot 4$	$z=6+2\cdot 0$	$z=10+2\cdot 0$
Total	10	6	10

Per tant aquest valor màxim s'assoleix en tots els punts compresos entre el vèrtex A i el C.•

Si volguéssim fer la resolució gràfica (no la demanen):

$$z = x+2y \;\rightarrow\; 0 = x+2y \;\rightarrow\; y = -\frac{1}{2}x,\ \textit{la representem i fem paral·leles pels vèrtexs}$$



3.-a)

$$f(x) = \frac{2x^2}{ax+1}$$

$$f'(x) = \frac{4x\cdot(ax+1)-2x^2\cdot a}{(ax+1)^2} = \frac{2ax^2+4x}{(ax+1)^2} \;\rightarrow\; f'(1) = \frac{2a\cdot 1^2+4\cdot 1}{(a\cdot 1+1)^2} = 0 \;\rightarrow$$

$$\rightarrow\; 2a+4=0 \Rightarrow \underline{a=-2}\bullet$$

$$f'(x)=\frac{2\cdot(-2)x^2+4x}{(-2x+1)^2}=\frac{-4x^2+4x}{(-2x+1)^2}$$

$$f''(x)=\frac{(-8x+4)\cdot(-2x+1)^2-(-4x^2+4x)\cdot 2\cdot(-2x+1)\cdot(-2)}{(-2x+1)^4} \rightarrow f''(1)=\frac{(-)\cdot(+)-0}{(+)}<0$$

Es tracta d'un màxim •

b)

$$f(x)=\frac{2x^2}{ax+1} \rightarrow f(x)=\frac{2x^2}{3x+1} \rightarrow 3x+1=0 \rightarrow x=-\frac{1}{3}\ \textit{discontínua.}$$

Asímptota vertical:

$$\lim_{x\to-\frac{1}{3}} f(x):\left\{\begin{array}{l}\lim\limits_{x\to-\frac{1}{3}^-} f(x)=\frac{2/9}{0^-}=-\infty \\ \lim\limits_{x\to-\frac{1}{3}^+} f(x)=\frac{2/9}{0^+}=+\infty\end{array}\right\}\ \textit{tenim asímptota vertical en } x=-\frac{1}{3}\ \bullet$$

Asímptota horitzontal:

$$\lim_{x\to\pm\infty} f(x)=\frac{+\infty}{\pm\infty}=\textit{Indeterminat, com grN > grD} \rightarrow \lim_{x\to\pm\infty} f(x)=\pm\infty \rightarrow$$

No tenim asímptotes horitzontals.

4.-

$$A=\begin{pmatrix}-1 & 2 & 2\\ 2 & 1 & -1\end{pmatrix}$$

a)

$$\begin{pmatrix}-1 & 2 & 2\\ 2 & 1 & -1\end{pmatrix}\cdot\begin{pmatrix}1 & 0\\ x & y\\ z & t\end{pmatrix}=\begin{pmatrix}5 & -2\\ 3 & -5\end{pmatrix} \Rightarrow \begin{cases}-1+2x+2z=5\\ 2+x-z=3\\ 2y+2t=-2\\ y-t=-5\end{cases} \rightarrow \left.\begin{array}{l}\begin{cases}-1+2x+2z=5\\ 2+x-z=3\end{cases}\\ \begin{cases}2y+2t=-2\\ y-t=-5\end{cases}\end{array}\right\} \rightarrow$$

$$\left.\begin{array}{l}\begin{cases}2x+2z=6\\ x-z=1\end{cases}\\ \begin{cases}2y+2t=-2\\ y-t=-5\end{cases}\end{array}\right\} \xrightarrow{\textit{reducció}} \begin{cases}4x=8\rightarrow x=2\rightarrow z=1\\ 4y=-12\rightarrow y=-3\rightarrow t=2\end{cases} \Rightarrow B=\begin{pmatrix}1 & 0\\ 2 & -3\\ 1 & 2\end{pmatrix}\ \bullet$$

b)

$$(A\cdot B)^t=\begin{pmatrix}5 & -2\\ 3 & -5\end{pmatrix}^t=\begin{pmatrix}5 & 3\\ -2 & -5\end{pmatrix}$$

$$A^t = \begin{pmatrix} -1 & 2 & 2 \\ 2 & 1 & -1 \end{pmatrix}^t = \begin{pmatrix} -1 & 2 \\ 2 & 1 \\ 2 & -1 \end{pmatrix} \qquad B^t = \begin{pmatrix} 1 & 0 \\ 2 & -3 \\ 1 & 2 \end{pmatrix}^t = \begin{pmatrix} 1 & 2 & 1 \\ 0 & -3 & 2 \end{pmatrix}$$

$$B^t \cdot A^t = \begin{pmatrix} 1 & 2 & 1 \\ 0 & -3 & 2 \end{pmatrix} \cdot \begin{pmatrix} -1 & 2 \\ 2 & 1 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} -1+4+2 & 2+2-1 \\ 0-6+4 & 0-3-2 \end{pmatrix} = \begin{pmatrix} 5 & 3 \\ -2 & -5 \end{pmatrix}$$

5.-

$$\left.\begin{array}{ll} Costos & C(x) = 180x + 12000 \\ Ingressos & I(x) = 500x - \frac{1}{2}x^2 \end{array}\right\} \rightarrow B(x) = I(x) - C(x) = 500x - \frac{1}{2}x^2 - (180x + 12000)$$

$$\rightarrow \quad B(x) = -\frac{1}{2}x^2 + 320x - 12000$$

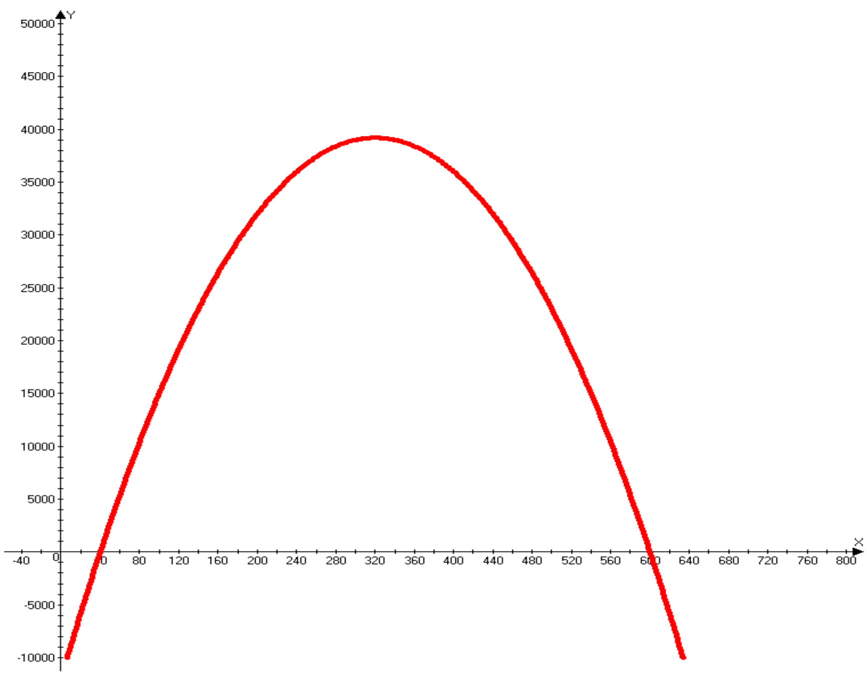
a)

No perdre diners vol dir que els beneficis no poden ser negatius, estudiem el signe de la paràbola:

$$-\frac{1}{2}x^2 + 320x - 12000 = 0 \quad \rightarrow \quad x = \frac{-320 \pm \sqrt{320^2 - 24000}}{-1} = \frac{-320 \pm 280}{-1} = \begin{cases} 40 \\ 600 \end{cases}$$

Al ser la funció una paràbola còncava, és positiva entre 40 *i* 600 $\Rightarrow$

$$\Rightarrow \quad B(x) \geq 0 \quad \Rightarrow \quad \forall x \in [40{,}600] \bullet$$



b)

$$y = -\frac{1}{2}x^2 + 320x - 12000 \quad \rightarrow \quad y' = -x + 320$$

$$\rightarrow \quad y' = 0 \quad \Rightarrow \quad -x + 320 = 0 \quad \rightarrow x = 320 \bullet$$

$$\text{Benefici per cada bicicleta} : \frac{B(x)}{x}$$

$$\frac{B(x)}{x} = \frac{-\frac{1}{2}(320)^2 + 320 \cdot (320) - 12000}{320} = \frac{39200}{320} = \underline{\underline{122'5}} \bullet$$

6.-

$$f(x) = x - e^{-3x}$$

a)

$$f(x) = x - e^{-3x} = x - \frac{1}{e^{3x}}$$

$$\underline{D(f) = R \ \text{ja que } e^{3x} \neq 0, \ \forall x \in R.}$$

$$f'(x) = 1 - e^{-3x} \cdot (-3) = 1 + 3 \cdot e^{-3x} > 0 \ , \ \forall x \in R \quad \rightarrow \quad \text{Estrictament creixent.}$$

b)

$$f(x) = x - e^{-3x} \quad \rightarrow \quad \left\{ \begin{array}{l} f(0) = 0 - e^{-3 \cdot 0} = -1 \rightarrow \text{punt de tangència } (0,-1) \\ f'(x) = 1 + 3 \cdot e^{-3x} \rightarrow f'(0) = 1 + 3 \cdot e^{-3 \cdot 0} = 4 \ \text{pendent} \end{array} \right\} \rightarrow$$

$$\rightarrow \quad y - (-1) = 4 \cdot (x - 0) \quad \rightarrow \quad \underline{\underline{y = 4x - 1}} \bullet$$

La representació del problema (no la demanen):