1.-

$$r:\begin{cases}2x-y+3z=2\\ x+z+1=0\end{cases}$$

a)

$$\vec{v_r}=(2,-1,3)\wedge(1,0,1)=\begin{vmatrix} i & 2 & 1\\ j & -1 & 0\\ k & 3 & 1\end{vmatrix}=-i+3j+0k+k-0i-2j=\underline{\underline{(-1,1,1)_{<i,j,k>}}}\bullet$$

b)

$$s\| r\ \rightarrow\ \vec{v_s}\|\vec{v_r}\ \Rightarrow\ \vec{v_s}=(-1,1,1)\ \rightarrow\ \underline{\underline{s:\frac{x-1}{-1}=\frac{y}{1}=\frac{z-1}{1}}}\bullet$$

2.-

$$X\cdot A+B=C$$

a)

$$X\cdot A+B=C\ \rightarrow\ X\cdot A=C-B\ \rightarrow\ \underline{\underline{X=(C-B)\cdot A^{-1}}}\bullet$$

b)

$$X=(C-B)\cdot A^{-1}=\left[\begin{pmatrix}3&1\\1&-1\end{pmatrix}-\begin{pmatrix}1&1\\-2&1\end{pmatrix}\right]\cdot\begin{pmatrix}1&-2\\-1&1\end{pmatrix}^{-1}=\begin{pmatrix}2&0\\3&-2\end{pmatrix}\cdot\begin{pmatrix}1&-2\\-1&1\end{pmatrix}^{-1}=*$$

$$\begin{pmatrix}1&-2\\-1&1\end{pmatrix}^{-1}:\ |A|=1-2=-1,\ A^T=\begin{pmatrix}1&-1\\-2&1\end{pmatrix}\ \rightarrow A^{-1}=\frac{1}{-1}\begin{pmatrix}1&+2\\+1&1\end{pmatrix}=\begin{pmatrix}-1&-2\\-1&-1\end{pmatrix}$$

$$*=\begin{pmatrix}2&0\\3&-2\end{pmatrix}\cdot\begin{pmatrix}-1&-2\\-1&-1\end{pmatrix}=\underline{\underline{\begin{pmatrix}-2&-4\\-1&-2\end{pmatrix}}}\bullet$$

3.-

$$f(x)=a\cdot(1-x^2)\ i\ g(x)=\frac{x^2-1}{a}\quad a>0.$$

a)

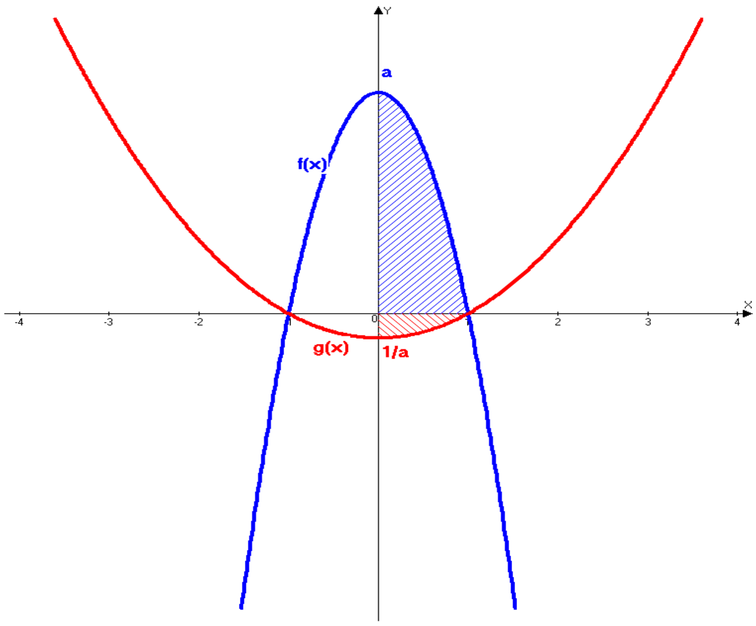
$$f(x)=a\cdot(-x^2+1)\ i\ g(x)=\frac{1}{a}\cdot(x^2-1)\ amb\ \ a>0$$

$$Punts\ de\ tall:\quad f(x)=g(x)\ \Rightarrow\ a\cdot(-x^2+1)=\frac{1}{a}\cdot(x^2-1)\ \rightarrow\ -a^2x^2+a^2=x^2-1$$

$$\rightarrow\ -a^2x^2-x^2=-1-a^2\ \rightarrow\ x^2\cdot(-a^2-1)=-1-a^2\ \rightarrow\ x^2=1\ \rightarrow x=\pm1$$

$$i\ a\ més\ \ f(\pm1)=g(\pm1)=0.$$

$$\textit{Tenim dues paràboles, una còncava i una altra convexa, que tenen per zeros } x = \pm 1$$



$$2\int_0^1 [f(x)+(-g(x))]dx = 2\int_0^1 [f(x)-g(x)]dx = 2\cdot\int_0^1 \left[a\cdot(1-x^2)-\frac{1}{a}\cdot(x^2-1)\right]dx =$$

$$= 2\cdot\int_0^1 \left[a\cdot(1-x^2)+\frac{1}{a}\cdot(-x^2+1)\right]dx = 2\cdot\int_0^1 \left(a+\frac{1}{a}\right)\cdot(-x^2+1)\cdot dx =$$

$$= 2\cdot\left(a+\frac{1}{a}\right)\cdot\left[-\frac{x^3}{3}+x\right]_0^1 = 2\cdot\left(a+\frac{1}{a}\right)\cdot\left(-\frac{1}{3}+1\right) = 2\cdot\left(a+\frac{1}{a}\right)\cdot\frac{2}{3} = \underline{\underline{\frac{4}{3}\cdot\left(\frac{a^2+1}{a}\right)}}\bullet$$

b)

$$\textit{Superfície en funció de } a: \quad S(a) = \frac{4}{3}\cdot\left(\frac{a^2+1}{a}\right)$$

$$S'(a) = \frac{4}{3}\cdot\frac{2a\cdot a-(a^2+1)\cdot 1}{a^2} = \frac{4}{3}\cdot\frac{a^2-1}{a^2} \quad\rightarrow\quad S'(a)=0 \quad\Rightarrow\quad a^2-1=0 \quad\rightarrow\quad a=\pm 1$$

$$S''(a) = \frac{4}{3}\cdot\frac{2a\cdot a^2-(a^2-1)\cdot 2a}{a^4} \quad\rightarrow\begin{cases} S''(1) = \dfrac{(+)-0}{(+)} > 0 \quad\Rightarrow\quad \underline{\underline{\textit{mínim}}}\bullet \\ S''(-1) = \dfrac{(-)-0}{(+)} < 0 \quad\rightarrow\quad \textit{màxim} \end{cases}$$

4.-

$$\begin{cases} x+2y-az=-3 \\ 2x+(a-5)y+z=4a+2 \\ 4x+(a-1)y-3z=4 \end{cases}$$

a)

$$(A\vdots\overline{A}) = \begin{pmatrix} 1 & 2 & -a & \vdots & -3 \\ 2 & a-5 & 1 & \vdots & 4a+2 \\ 4 & a-1 & -3 & \vdots & 4 \end{pmatrix}$$

$$\det A = \begin{vmatrix} 1 & 2 & -a \\ 2 & a-5 & 1 \\ 4 & a-1 & -3 \end{vmatrix} = -3a+15-2a^2+2a+8+4a^2-20a-a+1+12 = 2a^2-22a+36$$

$$2a^2-22a+36=0 \quad \rightarrow \quad a^2-11a+18=0 \quad \rightarrow \begin{cases} a=2 \\ a=9 \end{cases}$$

$$* \ Si \ a \neq 2 \ i \ a \neq 9 \quad \rightarrow \quad \det A \neq 0 \quad \rightarrow rangA = rang\,\overline{A} = 3 \quad \rightarrow C..D.(no \ interessa)$$

$$* \ \underline{\underline{Si \ a=2 \ o \ a=9: \ \det A = 0 \quad \Rightarrow \quad rangA<3}} \ no \ pot \ ser \ C.D. \bullet$$

$$** \ a=2$$

$$\begin{pmatrix} 1 & 2 & -2 & \vdots & -3 \\ 2 & -3 & 1 & \vdots & 10 \\ 4 & 1 & -3 & \vdots & 4 \end{pmatrix} \quad \rightarrow \begin{vmatrix} 1 & 2 \\ 2 & -3 \end{vmatrix} \neq 0 \quad \rightarrow rangA = 2 \qquad \begin{vmatrix} 1 & 2 & -3 \\ 2 & -3 & 10 \\ 4 & 1 & 4 \end{vmatrix} = 0$$

$$i \ com \ C_3 = -C_2 \quad \rightarrow \quad tots \ els \ menors \ d'ordre \ 3 \ de \ \overline{A} \ tenen \ determinant \ zero \rightarrow$$

$$, \quad rang\,\overline{A} = 2 \qquad COMPATIBLE \quad INDETERMINAT.$$

b)

$$\begin{cases} x+2y-az=-3 \\ 2x+(a-5)y+z=4a+2 \\ 4x+(a-1)y-3z=4 \end{cases} \quad \rightarrow \begin{cases} 1-6+a=-3 \\ 2-(a-5)3-1=4a+2 \\ 4-(a-1)3+3=4 \end{cases} \quad \rightarrow \begin{cases} a=2 \\ a=2 \\ a=2 \end{cases}$$

$$però \ per \ aquest \ valor \ de \ a, el \ sitema \ és \ compatible \ indeterminat, per \ tant$$

$$la \ solució \ donada \ no \ pot \ ser \ solució \ ùnica. \quad \rightarrow \quad \underline{\underline{no \ existeix \ tal \ valor \ de \ a.}} \bullet$$

5.-

$$r_1: \frac{x-2}{1} = \frac{y-3}{2} = \frac{z-1}{-2} \qquad r_2: \frac{x+3}{2} = \frac{y+1}{1} = \frac{z+1}{2}$$

a)

$$\overrightarrow{v_{r1}} \cdot \overrightarrow{v_{r2}} = (1,2,-2)\cdot(2,1,2) = 2+2-4=0 \quad \rightarrow \quad \overrightarrow{v_{r1}} \perp v_{r2} \quad \rightarrow r_1 \perp r_2 \bullet$$

b)

$$r_1: \frac{x-2}{1} = \frac{y-3}{2} = \frac{z-1}{-2} \quad \rightarrow \begin{cases} 2x-4=y-3 \\ -2x+4=z-1 \end{cases}$$

$$r_2: \frac{x+3}{2} = \frac{y+1}{1} = \frac{z+1}{2} \quad \rightarrow \begin{cases} x+3=2y+2 \\ z+1=2y+2 \end{cases}$$

$$r_1 \cap r_2: \begin{cases} 2x-1=y \\ -2x+5=z \\ \dfrac{x+1}{2}=y \\ z=2y+1 \end{cases} \quad \rightarrow \quad 2x-1=\frac{x+1}{2} \quad \rightarrow \quad \underline{\underline{x=1 \quad \rightarrow y=1 \quad z=3}} \bullet$$

6.-

$$f(x) = x^2 \cdot e^{-ax}\ ,\ a \neq 0$$

a)

$$f(x) = x^2 \cdot e^{-ax}$$

$$f'(x) = 2x \cdot e^{-ax} + x^2 \cdot e^{-ax}(-a) = 2x \cdot e^{-ax} - a \cdot x^2 \cdot e^{-ax} = e^{-ax} \cdot \left(2x - ax^2\right)$$

$$f'(2) = e^{-a2} \cdot (2 \cdot 2 - a \cdot 4) = 0 \quad \rightarrow \quad 2 \cdot 2 - a \cdot 4 = 0 \quad \rightarrow \quad \underline{\underline{a = 1}} \bullet$$

b)

$$f(x) = x^2 \cdot e^{-2x}$$

$$f'(x) = 2x \cdot e^{-2x} + x^2 \cdot e^{-2x}(-2) = 2x \cdot e^{-2x} - 2 \cdot x^2 \cdot e^{-2x} = e^{-2x} \cdot \left(2x - 2x^2\right) = 0$$

$$\rightarrow \quad 2x - 2x^2 = 0 \quad \rightarrow 2x \cdot (1 - x) = 0 \quad \rightarrow \quad \begin{cases} x = 0 \\ x = 1 \end{cases}$$

$$f''(x) = e^{-2x} \cdot (-2) \cdot \left(2x - 2x^2\right) + e^{-2x} \cdot (2 - 4x)$$

$$\begin{cases} f''(0) = e^0 \cdot (-2) \cdot 0 + e^0 \cdot (2 - 0) = 0 + 2 > 0 \quad \rightarrow \underline{\underline{(0,0)\ mínim}} \bullet \\ f''(1) = e^{-2} \cdot (-2) \cdot 0 + e^{-2} \cdot (2 - 4) = \dfrac{1}{e^2}(-2) < 0 \rightarrow \underline{\underline{\left(1, \tfrac{1}{e^2}\right) màxim}} \bullet \end{cases}$$