1.-

$$Com\ estem\ a\ R^3,\ si\ s'han\ de\ tallar\ en\ una\ recta,\ el\ sistema\ ha\ de\ ser:$$

$$\left.\begin{array}{l} COMPATIBLE \leftrightarrow es\ tallen \\ INDETERMINAT\ AMB\ GRAU\ DE\ LLIBERTAT\ 1 \leftrightarrow en\ una\ recta \end{array}\right\} \leftrightarrow rangA = rang\overline{A} = 2$$

$$\left.\begin{array}{l} x-y+mz=1 \\ x-y+z=m \\ my+2z=3 \end{array}\right\} \rightarrow \left(A|\overline{A}\right) = \left(\begin{array}{ccc|c} 1 & -1 & m & 1 \\ 1 & -1 & 1 & m \\ 0 & m & 2 & 3 \end{array}\right)$$

$$rangA = 2 \Rightarrow \det A = 0 \rightarrow \begin{vmatrix} 1 & -1 & m \\ 1 & -1 & 1 \\ 0 & m & 2 \end{vmatrix} = m^2 - m = 0 \rightarrow \begin{cases} m=0 \\ m=1 \end{cases}$$

- $m = 0$:

$$\left(A|\overline{A}\right) = \left(\begin{array}{ccc|c} 1 & -1 & 0 & 1 \\ 1 & -1 & 1 & 0 \\ 0 & 0 & 2 & 3 \end{array}\right) \rightarrow \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 2 & 3 \end{vmatrix} \neq 0 \rightarrow rang\overline{A} = 3\quad INCOMPATIBLE \Rightarrow NO!$$

- $\underline{\underline{m = 1}}$:

$$\left(A|\overline{A}\right) = \left(\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 0 & 0 & 2 & 3 \end{array}\right) \Rightarrow rang\overline{A} = 2\ \ ja\ que\ F1 = F2 \rightarrow \underline{\underline{COMPT.\ INDETER}} \bullet$$

2.-

a)

$$y = 3x + b\ \ recta\ tangent:\ pendent\ 3\ i\ rectes\ paral{\cdot}leles\ tenen\ el\ mateix\ pendent.$$

$$y = x^2 \rightarrow y' = 2x \Rightarrow 2x = 3 \rightarrow \underline{\underline{x = 3/2}} \bullet$$

b)

$$Si\ x = 3/2 \rightarrow y = \left(3/2\right)^2 = 9/4 \Rightarrow punt\ de\ tangència: \left(3/2, 9/4\right) \rightarrow$$

$$9/4 = 3\cdot 3/2 + b \rightarrow 9/4 = 9/2 + b \rightarrow 9/4 - 9/2 = b \rightarrow \underline{\underline{-9/2 = b}} \bullet$$

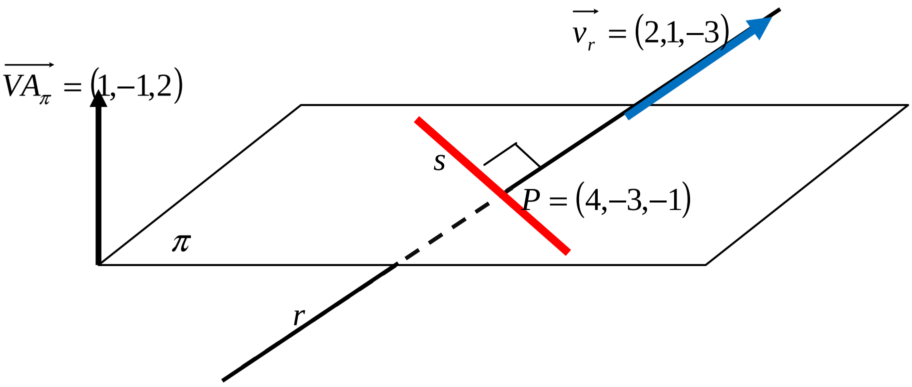
3.-a)

$$\left.\begin{array}{l} x-y+2z-5=0 \\ \begin{cases} x+y+z=0 \\ 2x-y+z=10 \end{cases} \end{array}\right\} \rightarrow \underline{x = 5+y-2z} \Rightarrow \begin{cases} 5+y-2z+y+z=0 \\ 10+2y-4zy-+z=10 \end{cases} \rightarrow$$

$$\rightarrow \left.\begin{array}{l} 5+2y-z=0 \\ y-3z=0 \end{array}\right\} \rightarrow \underline{y=3z} \Rightarrow 5+6z-z=0 \rightarrow \underline{\underline{z=-1, y=-3, x=4}} \bullet$$

$$\underline{\underline{P = (4,-3,-1)}}$$

b)



$$\overrightarrow{v_r}=\begin{vmatrix} i & 1 & 2\\ j & 1 & -1\\ k & 1 & 1\end{vmatrix}=i+2j-k-2k+i-j=2i+j-3k \quad\rightarrow\quad (2,1,-3)$$

$$s:\begin{Bmatrix} s\perp r\rightarrow \overrightarrow{v_s}\perp\overrightarrow{v_r}\\ s\subset\pi\rightarrow \overrightarrow{v_s}\perp\overrightarrow{VA_\pi}\end{Bmatrix}\quad\rightarrow\quad \overrightarrow{v_s}=\overrightarrow{v_r}\wedge\overrightarrow{VA_\pi}=\begin{vmatrix} i & 2 & 1\\ j & 1 & -1\\ k & -3 & 2\end{vmatrix}=2i-3j-2k-3i-4j-k=$$

$$=-i-7j-3k \quad\rightarrow\quad \overrightarrow{v_s}=(1,7,3)\bullet$$

$$s:\ \frac{x-4}{1}=\frac{y+3}{7}=\frac{z+1}{3}\bullet$$

4.-

a)

$$A=\begin{pmatrix}3 & 2\\ -1 & 1\end{pmatrix}\qquad B=\begin{pmatrix}1 & -2\\ 1 & 3\end{pmatrix}$$

$$A+B=\begin{pmatrix}3 & 2\\ -1 & 1\end{pmatrix}+\begin{pmatrix}1 & -2\\ 1 & 3\end{pmatrix}=\begin{pmatrix}4 & 0\\ 0 & 4\end{pmatrix}\qquad A-B=\begin{pmatrix}3 & 2\\ -1 & 1\end{pmatrix}-\begin{pmatrix}1 & -2\\ 1 & 3\end{pmatrix}=\begin{pmatrix}2 & 4\\ -2 & -2\end{pmatrix}$$

$$(A+B)\cdot(A-B)=\begin{pmatrix}4 & 0\\ 0 & 4\end{pmatrix}\cdot\begin{pmatrix}2 & 4\\ -2 & -2\end{pmatrix}=\begin{pmatrix}8 & 16\\ -8 & -8\end{pmatrix}\bullet$$

$$A^2=\begin{pmatrix}3 & 2\\ -1 & 1\end{pmatrix}\cdot\begin{pmatrix}3 & 2\\ -1 & 1\end{pmatrix}=\begin{pmatrix}7 & 8\\ -4 & -1\end{pmatrix}\qquad B^2=\begin{pmatrix}1 & -2\\ 1 & 3\end{pmatrix}\cdot\begin{pmatrix}1 & -2\\ 1 & 3\end{pmatrix}=\begin{pmatrix}-1 & -8\\ 4 & 7\end{pmatrix}$$

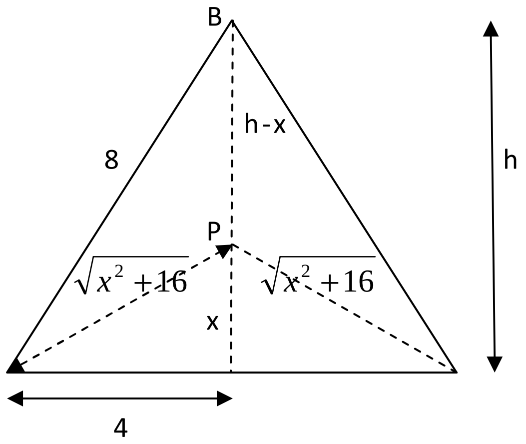
$$A^2-B^2=\begin{pmatrix}7 & 8\\ -4 & -1\end{pmatrix}-\begin{pmatrix}-1 & -8\\ 4 & 7\end{pmatrix}=\begin{pmatrix}8 & 16\\ -8 & -8\end{pmatrix}\bullet$$

b)

$$(A+B)\cdot(A-B)=A^2-A\cdot B+B\cdot A-B^2$$

$$(A+B)\cdot(A-B)=A^2-B^2\ \ si:\ -A\cdot B+B\cdot A=0\quad\Leftrightarrow\quad B\cdot A=A\cdot B,\quad A\ i\ B\ conmuten.$$

5.-



a) $h=\sqrt{64-16}=\sqrt{48}$

b) $d(P,A)=\sqrt{x^2+16}$ $\quad d(P,C)=\sqrt{x^2+16}$ $\quad d(P,B)=h-x=\sqrt{48}-x$

c)

$$D=\left(\sqrt{x^2+16}\right)^2+\left(\sqrt{x^2+16}\right)^2+\left(\sqrt{48}-x\right)^2$$

$$D=x^2+16+x^2+16+\left(\sqrt{48}-x\right)^2$$

$$D'=2x+2x+2\cdot\left(\sqrt{48}-x\right)\cdot(-1)=6x-2\sqrt{48}$$

$$D'=0 \quad\rightarrow\quad 6x-2\sqrt{48}=0 \quad\Rightarrow\quad \underline{\underline{x=\frac{2\sqrt{48}}{6}=\frac{2\sqrt{6}}{3}}} \quad\rightarrow\quad D''=6>0 \quad \textit{minim.}$$

6.-a)

$$Ax+By+Cz+D=0 \quad\rightarrow \begin{matrix} P(1,0,0) & \rightarrow & A\cdot 1+B\cdot 0+C\cdot 0+D=0 \\ Q(0,2,0) & \rightarrow & A\cdot 0+B\cdot 2+C\cdot 0+D=0 \\ R(0,0,3) & \rightarrow & A\cdot 0+B\cdot 0+C\cdot 3+D=0 \end{matrix}\Bigg\} \rightarrow$$

$$\rightarrow \left\{\begin{matrix} A=-D \\ B=-D/2 \\ C=-D/3 \end{matrix}\right\} \quad\Rightarrow\quad -Dx-\frac{D}{2}y-\frac{D}{3}z+D=0 \quad\rightarrow\quad 6Dx+3Dy+2Dz-6D=0$$

$$\rightarrow \quad \underline{\underline{6x+3y+2z-6=0}}\bullet$$

b)

$S(1,2,3)$ *verifica l'equació del pla construit?*

$$6\cdot 1+3\cdot 2+2\cdot 3-6\overset{?}{=}0 \quad\rightarrow\quad 12\neq 0 \quad \textit{no són coplanaris.}$$