RESOLUCIÓ

1.- $$f(x)=\frac{a}{x^2+bx+c}$$

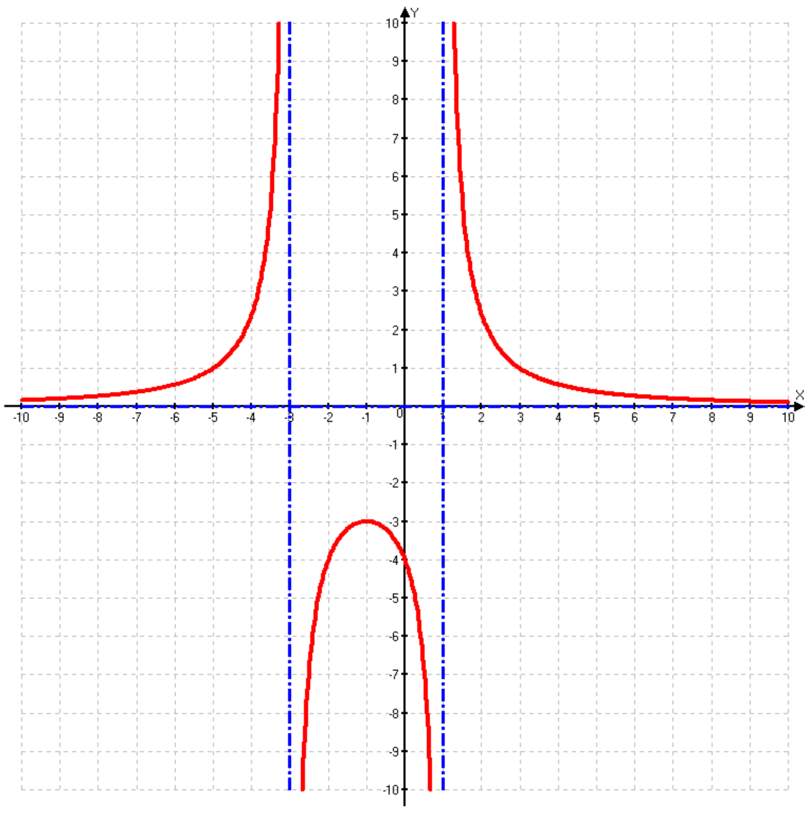
a)

$$\left.\begin{array}{l}\bullet\ AV\begin{cases}x=-3 \;\rightarrow\; (x^2+bx+c)_{x=-3}=0 \;\rightarrow\; 9-3b+c=0\\ x=1 \;\rightarrow\; (x^2+bx+c)_{x=1}=0 \;\rightarrow\; 1+b+c=0\end{cases}\\ \bullet\ Passa\ \ pel\ \ punt\ (0,-4)\rightarrow\ -4=\dfrac{a}{0^2+0b+c}\ \rightarrow\ -4c=a\end{array}\right\}\rightarrow\ \underline{\underline{c=3b-9}}\ \Rightarrow$$

$$\Rightarrow\ \begin{Bmatrix}1+b+3b-9=0\\ -4c=a\end{Bmatrix}\ \rightarrow\ \underline{\underline{b=2}}\bullet\ \rightarrow\ c=3\cdot 2-9\ \rightarrow\ \underline{\underline{c=-3}}\bullet\ \rightarrow\ \underline{\underline{a=12}}\bullet$$

$$f(x)=\frac{12}{x^2+2x-3}\ \rightarrow\ té\ \ A.H.\ \ y=0,\ \ ja\ que\ \ \lim_{x\to\infty}\left(\frac{12}{x^2+2x-3}\right)=\frac{12}{\infty}=0$$

$$f(x)>0,\ \ \forall x\in(-\infty,-3)$$
$$f(x)<0,\ \ \forall x\in(-3,1)$$
$$f(x)>0,\ \ \forall x\in(1,\infty)$$



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b)

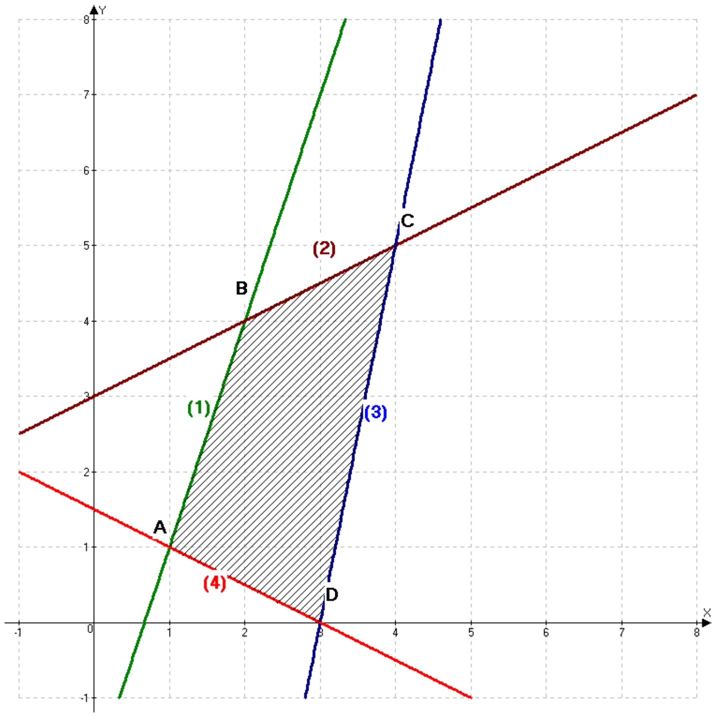
$$\left.\begin{aligned} a &= 1 \\ b &= -2 \\ c &= -1 \end{aligned}\right\} \rightarrow \quad f(x) = \frac{1}{x^2 - 2x - 1} \quad \rightarrow \quad f'(x) = \frac{0 - (2x-2)}{(x^2 - 2x - 1)^2}$$

$$f'(x) = 0 \quad \rightarrow \quad \frac{0-(2x-2)}{(x^2-2x-1)^2} = 0 \quad \rightarrow \quad 2x - 2 = 0 \quad \rightarrow \quad x = 1$$

$$f''(x) = \frac{-2\cdot(x^2-2x-1)^2 - [-(2x-2)]\cdot 2\cdot(x^2-2x-1)\cdot(2x-2)}{(x^2-2x-1)^4} \quad \rightarrow$$

$$f''(1) = \frac{(-)\cdot(+) - 0}{(+)} < 0 \quad \Rightarrow \quad \textit{Màxim en el punt} \left\{\begin{aligned} & x = 1 \\ & f(1) = \frac{1}{1^2 - 2\cdot 1 - 1} = -\frac{1}{2} \end{aligned}\right\} \left(1, -\frac{1}{2}\right) \bullet$$

2.-a)



$$(1) \quad y = mx + n \quad \rightarrow \quad \left\{\begin{aligned} (1,1) &\rightarrow 1 = m\cdot 1 + n \\ (2,4) &\rightarrow 4 = m\cdot 2 + n \end{aligned}\right\} \xrightarrow{\text{Reducció}} \left\{\begin{aligned} m &= 3 \\ n &= -2 \end{aligned}\right\} \rightarrow y = 3x - 2 \quad \Rightarrow y \le 3x - 2 \bullet$$

$$(2) \quad y = mx + n \quad \rightarrow \quad \left\{\begin{aligned} (2,4) &\rightarrow 4 = m\cdot 2 + n \\ (4,5) &\rightarrow 5 = m\cdot 4 + n \end{aligned}\right\} \xrightarrow{\text{Reducció}} \left\{\begin{aligned} m &= 1/2 \\ n &= 3 \end{aligned}\right\} \rightarrow y = \frac{1}{2}x + 3 \quad \Rightarrow y \le \frac{1}{2}x + 3 \bullet$$

$$(3) \quad y = mx + n \quad \rightarrow \quad \left\{\begin{aligned} (3,0) &\rightarrow 0 = m\cdot 3 + n \\ (4,5) &\rightarrow 5 = m\cdot 4 + n \end{aligned}\right\} \xrightarrow{\text{Reducció}} \left\{\begin{aligned} m &= 5 \\ n &= -15 \end{aligned}\right\} \rightarrow y = 5x - 15 \quad \Rightarrow y \ge 5x - 15 \bullet$$

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$$(4)\quad y = mx+n \quad \to \quad \begin{cases} (3,0) \to 0 = m\cdot 3 + n \\ (1,1) \to 1 = m\cdot 1 + n \end{cases} \xrightarrow{\text{Reducció}} \begin{cases} m = -\dfrac{1}{2} \\ n = \dfrac{3}{2} \end{cases} \to y = -\frac{1}{2}x + \frac{3}{2} \Rightarrow y \geq -\frac{1}{2}x + \frac{3}{2} \bullet$$

$$\left. \begin{array}{l} y \leq 3x - 2 \\ y \leq \dfrac{1}{2}x + 3 \\ y \geq 5x - 15 \\ y \geq -\dfrac{1}{2}x + \dfrac{3}{2} \end{array} \right\} \leftrightarrow \underline{\underline{\begin{cases} 0 \leq 3x - y - 2 \\ 0 \leq x - 2y + 6 \\ 0 \geq 5x - y - 15 \\ 0 \geq -x - 2y + 3 \end{cases}}} \bullet$$

b)

$$P(3,1) \to \begin{cases} 0 \overset{?}{\leq} 3\cdot 3 - 1 - 2 & \to Veritat \\ 0 \overset{?}{\leq} 3 - 2\cdot 1 + 6 & \to Veritat \\ 0 \overset{?}{\geq} 5\cdot 3 - 1 - 15 & \to Veritat \\ 0 \overset{?}{\geq} -3 - 2\cdot 1 + 3 & \to Veritat \end{cases} \Rightarrow \textit{Interior a la regió.}$$

$$Q(3,4) \to \begin{cases} 0 \overset{?}{\leq} 3\cdot 3 - 4 - 2 & \to Veritat \\ 0 \overset{?}{\leq} 3 - 2\cdot 4 + 6 & \to Veritat \\ 0 \overset{?}{\geq} 5\cdot 3 - 4 - 15 & \to Veritat \\ 0 \overset{?}{\geq} -3 - 2\cdot 4 + 3 & \to Veritat \end{cases} \Rightarrow \textit{Interior a la regió.}$$

$$R(5,2) \to \begin{cases} 0 \overset{?}{\leq} 3\cdot 5 - 2 - 2 & \to Veritat \\ 0 \overset{?}{\leq} 5 - 2\cdot 2 + 6 & \to Veritat \\ 0 \overset{?}{\geq} 5\cdot 5 - 2 - 15 & \to \underline{Fals} \\ 0 \overset{?}{\geq} -3 - 2\cdot 1 + 3 & \to Veritat \end{cases} \Rightarrow \textit{Exterior a la regió.}$$

3.-a)

$$A = \begin{pmatrix} 2 & 1 & 3 \\ -1 & 0 & 4 \end{pmatrix} \quad B = \begin{pmatrix} 1 & -1 \\ 0 & 5 \end{pmatrix}$$

$$A\cdot B = \begin{pmatrix} 2 & 1 & 3 \\ -1 & 0 & 4 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 \\ 0 & 5 \end{pmatrix} = \textit{no es pot operar, sobra una columna a A o falta una fila a B}$$

$$B\cdot A = \begin{pmatrix} 1 & -1 \\ 0 & 5 \end{pmatrix} \cdot \begin{pmatrix} 2 & 1 & 3 \\ -1 & 0 & 4 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 3 & 1 & -1 \\ -5 & 0 & 20 \end{pmatrix}}} \bullet$$

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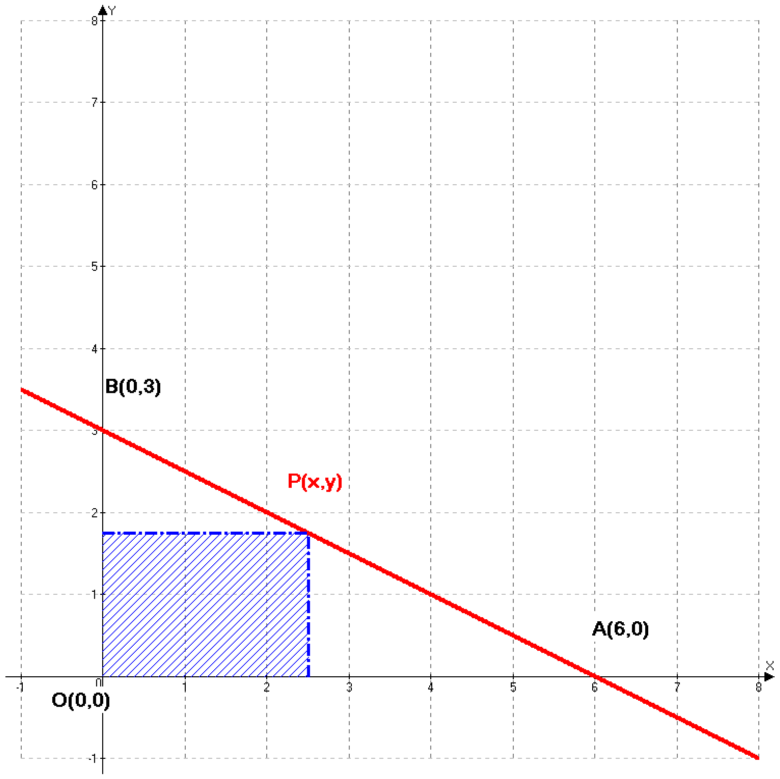
b)

$$B=\begin{pmatrix}1 & -1\\ 0 & 5\end{pmatrix}$$

$$B^2=B\cdot B=\begin{pmatrix}1 & -1\\ 0 & 5\end{pmatrix}\cdot\begin{pmatrix}1 & -1\\ 0 & 5\end{pmatrix}=\underline{\underline{\begin{pmatrix}1 & -6\\ 0 & 25\end{pmatrix}}}\bullet$$

$$B^3=B^2\cdot B=\begin{pmatrix}1 & -6\\ 0 & 25\end{pmatrix}\cdot\begin{pmatrix}1 & -1\\ 0 & 5\end{pmatrix}=\underline{\underline{\begin{pmatrix}1 & -31\\ 0 & 75\end{pmatrix}}}\bullet$$

4.-a)



$$y=mx+n \quad \rightarrow \begin{cases}(0,3)\rightarrow 3=m\cdot 0+n\\ (6,0)\rightarrow 0=m\cdot 6+n\end{cases} \xrightarrow{\text{Reducció}} \begin{cases} m=-\frac{1}{2}\\ n=3\end{cases} \rightarrow \underline{\underline{y=-\frac{1}{2}x+3}}\bullet$$

b)

$$S=x\cdot y=x\cdot\left(-\frac{1}{2}x+3\right)=-\frac{1}{2}x^2+3x \quad\rightarrow\quad S'=-x+3 \quad\rightarrow\quad S'=0\Rightarrow x=3\rightarrow$$

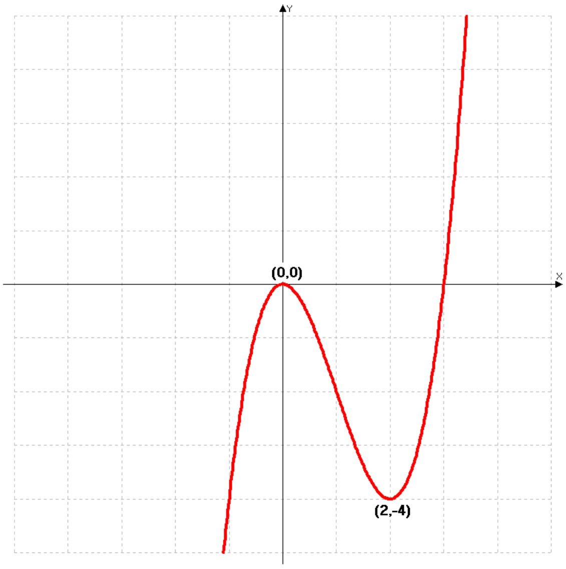
$$\rightarrow y=-\frac{1}{2}\cdot 3+3=\frac{3}{2}.$$

$$S''=-3<0 \quad\rightarrow\quad \textit{tenim un màxim}.\ \underline{\underline{P\left(3,\frac{3}{2}\right)}}\bullet$$

RESOLUCIÓ

5.-

a) *Una funció polinòmica és una funció contínua* :



b)

$$f(x)=ax^3+bx^2+cx+d \quad \rightarrow \quad f'(x)=3ax^2+2bx+c$$

$$(0,0)\rightarrow\left\{\begin{array}{lll} f(0)=0 & \rightarrow & 0=a\cdot 0^3+b\cdot 0^2+c\cdot 0+d \\ f'(0)=0 & \rightarrow & 0=3a\cdot 0^2+2b\cdot 0+c \end{array}\right\}\Rightarrow \underline{\underline{c=d=0}}\bullet$$

$$(2,-4)\rightarrow\left\{\begin{array}{lll} f(2)=-4 & \rightarrow & -4=a\cdot 2^3+b\cdot 2^2+c\cdot 2+d \\ f'(2)=0 & \rightarrow & 0=3a\cdot 2^2+2b\cdot 2+c \end{array}\right\}\rightarrow \left.\begin{array}{l} -4=8a+4b \\ 0=12a+4b \end{array}\right\}\Rightarrow \underline{\underline{\left\{\begin{array}{l} a=1 \\ b=-3 \end{array}\right.}}\bullet$$

$$\underline{\underline{f(x)=x^3-3x^2}}\bullet$$

6.-

$$\left.\begin{array}{l} Joan\rightarrow x \\ Pere\rightarrow y \\ Marc\rightarrow z \end{array}\right\}\rightarrow\left\{\begin{array}{l} x+y+z=63 \\ x-3=2\cdot(y+z) \\ y+1=\frac{1}{2}z \end{array}\right\}\rightarrow \underline{2y+2=z}\Rightarrow\left\{\begin{array}{l} x+3y=61 \\ x-3=2\cdot(3y+2) \end{array}\right\}\rightarrow$$

$$\rightarrow \left.\begin{array}{l} x+3y=61 \\ x-6y=7 \end{array}\right\}\rightarrow \underline{x=7+6y}\Rightarrow 9y=54 \rightarrow \underline{\underline{y=6,\ x=43,\ z=14}}\bullet$$